Electromagnetic Standard Fields: Generation and Accuracy Levels from 100 KHz to 990 MHz

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***Abstract* —The interest for problems concerning health protection against RF and MW electromagnetic fields is increasing more and more. This requires uniformity in performances supplied by the measurement instruments employed. It follows the importance of accurately evaluating the procedures of field generation and the overall indetermination on the obtainable field strength levels as well as the need of an intercomparison, carried out by means of a traveling standard too, between the laboratories operating in different countries. In this way, the need for standardizing the exposures techniques in the ever increasing number of experiments addressed to the study of biological effects is satisfied, too. For this purpose, an instrumental chain is described. This chain allows the generation of standard electromagnetic fields, in the context of the Italian National Health Service, with frequencies ranging from 100 KHz to 990 MHz and with field strength levels superior to the limits reported in recent international guidelines. Finally, the overall indetermination of the reached field strength levels is evaluated and discussed.**

I. Introduction

According to present guidelines, a correct realization of the health protection of workers and the general public against the RF and MW electromagnetic fields requires uniformity in performances supplied. For this purpose, the generation of standard electromagnetic fields is necessary, the spatial configuration of which has to be known with a great accuracy. The attainment of this aim meets as well the requirement of standardizing the exposure methods to the increased number of widespread and deepened experiments addressed to the thermal and nonthermal dosimetry and to the analysis of the consequent effects on animals and biological samples.

Our laboratory, responsible for the studies and research in order to attain adequate health and safety for people exposed to RF electromagnetic fields, realized the importance of analyzing the means of the laboratory generation and control of these fields so as to fully satisfy the requirements connected with the abovementioned problems.

The aim of this work is to describe procedures and methods adopted for the evaluation of both the electromagnetic-field intensities and the accuracy levels.

II. Instruments and Methods

The electromagnetic field in the frequency range from 100 KHz to 990 MHz is produced by a synthesized signal generator, amplified by solid-state amplifiers, and sent to a transverse electromagnetic (TEM) cell [1] for frequencies ranging from 100 KHz to 250 MHz, and to a directive antenna (double-ridged horn) placed in a shielded anechoic chamber in the remaining frequency range from 250 to 990 MHz, respectively.

In this way, it is possible to obtain the maximum electric-field intensity level in the TEM cell between 250 and 300 V/m for frequencies ranging from 100 KHz to 250 MHz. The maximum power density obtainable in the anechoic chamber, at a distance of 1 m from the antenna aperture, varies between 40 and 55 W/m^2 for frequencies ranging from 250 to 990 MHz. Such values are superior to the limits for occupational exposure set by recent international guidelines [2].

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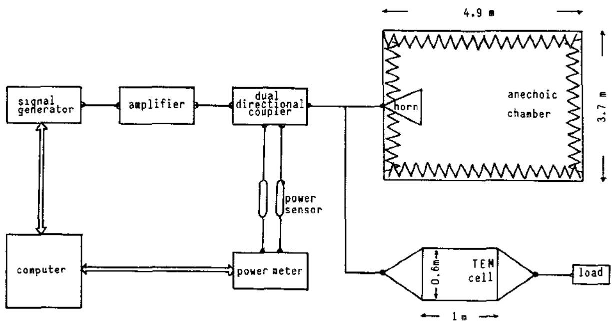

Fig. 1. Block diagram of the instrumental chain.

A bidirectional coupler has been placed between the amplifier and the TEM cell or the antenna.

This device allows the withdrawal of two signals proportional to the incident and reflected power, respectively. Such signals are sent to corresponding bolometric sensors that, connected to a power meter, allow the continuous monitoring of the available power to the load. A personal computer interfaced with the signal generator and with the power meter permits then the maintenance at constant values, even for long periods of time, of the field-intensity levels. Furthermore, this interfacing is useful in order to correlate field levels with frequencies for the pertinent written records. The scheme of the instrumental chain, set up at our laboratory, is reported in Fig. 1.

III. Field Strength Levels Evaluation

The electromagnetic-field level evaluation, known as the available power to the load P_a, is carried out in the TEM cell [1] by means of the following equation:

$$E = \sqrt{ZP_a}/d \tag{1}$$

where Z is the absolute value of the cell's complex characteristic impedance, and d is the distance between the cell's upper wall and its central plane.

In the anechoic chamber, the power density S is

$$S = P_a GK/4\pi R^2 \tag{2}$$

where G is the antenna gain, R is the distance in meters from the antenna, and K is a factor which accounts for the reflection. Such a reflection is evaluated comparing the experimental data, obtained by means of the insertion loss method with that theoretically forecasted by the transmission loss method, assuming that the theoretical prevision is exact [3], [4]. The power P_a is given by

$$P_a = P_i - P_r \tag{3}$$

where P_i, the power incident on the load, is

$$P_i = P_c(\alpha - 1)\beta\gamma \tag{4}$$

where P_c is the power meter reading proportional to the power incident upon the load, α is the direct coupling factor (i.e., the amount of incident power withdrawn by the bidirectional coupler and sent to the power sensor), β is the calibration factor of the bolometric sensor s, γ is the matching factor between the bidirectional coupler and the bolometric sensor, and P_r the power reflected by the load, is

$$P_r = P_d \alpha' \beta' \gamma \tag{5}$$

where P_d is the power meter reading proportional to the power reflected by the load, α' is the reverse coupling factor (i.e., the amount of reflected power withdrawn by the bidirectional coupler and sent to the power sensor), β' is the calibration factor of the bolometric sensor s', and γ is the matching factor mentioned above.

The gain G is experimentally determined by means of the "two standard antennas" method [5], [6], which allows the attainment of high accuracy levels. This method is based upon the use, as a receiving device, of an antenna identical to that under test.

The two antennas, referred to as A and B, respectively, must be perfectly aligned, polarization matched, and at a distance such that the wave incident upon the receiving antenna may be considered plane, that is, the measurement shall be carried out in far-field conditions.

In these circumstances, the absolute gain evaluation is based upon the Friis' formula [7]

$$P_B = P_A G_A G_B (\lambda / 4\pi R)^2 \tag{6}$$

where P_A is the power available to the antenna A, P_B is the power received by the antenna B, G_A is the gain of the antenna A, G_B is the gain of the antenna B, λ is the wavelength, and R is the distance between the two antennas.

As the two antennas are identical, the terms G_A and G_B are the same too. It follows that the experimental measurement refers to the square of the gain, therefore, the error relative to the gain will be halved. Since a continuous monitoring is not needed in order to accomplish this determination, the measurement of P_A is effected without the use of the bidirectional coupler, just substituting the antenna for the bolometric sensor. Such a procedure avoids the errors connected with the bidirectional coupler use, yet introduces an error due to the different mismatching between the generator and the load in both cases (antenna and power sensor).

The power P_B is determined directly connecting a bolometric sensor to the antenna output.

IV. Accuracy Levels

The accuracy levels that are possible to reach in order to know the value of the electromagnetic field generated in the anechoic chamber or in the TEM cell determine the reliability of the whole instrumental chain. Therefore, an exact estimate of the achievable accuracy levels becomes fundamental.

Such a valuation has been carried out by means of the "worst case method." This method, the more generally adopted, is based on the assumption that every error source act in the same direction with their maximum amplitude, so that an estimate widely reliable is obtained.

In the overall error determination, attention should be paid to the fact that the power sensor and the line under test are never perfectly matched, that is, their impedances are never exactly the same. This means that the power measured by the sensor is not actually that available to the line due to the reflection at the input of the sensor itself. Such a fact should be accounted for by means of a correction factor, the evaluation of which implies the measurement of the reflection coefficient. This reflection coefficient, a phasor, is given by

$$\Gamma = \rho e^{j\Phi} \tag{7}$$

where the modulus ρ for each component is generally supplied as datum or experimentally deducible. Such a quantity is always referred to the matching with an ideal 50-Ω impedance Z_0. The phase Φ depends on the frequency and varies with the distance along the line. Denoting $P_{i,j}$, the power supplied by the ith component to the jth component and with P_{i,Z_0}, the power that the ith component would supply to a standard impedance Z_0 component, the microwave system theory [8] assures that

$$\frac{P_i, Z_0}{P_{i,j}} = \frac{|1-\Gamma_i\Gamma_j|^2}{1-|\Gamma_j|^2} \tag{8}$$

where Γ_i and Γ_j are the ith and the jth element complex reflection coefficient respectively. Expressing (8) in decibels, we obtain

$$10\log(P_{i,Z_0}/P_{i,j}) = 10\log|1-\Gamma_i\Gamma_j|^2 - 10\log(1-|\Gamma_j|^2). \tag{9}$$

The second term of (9) depends only on the features of the jth component and is referred to as the mismatch loss. When the jth element is a power sensor, such a term is generally included in the power sensor calibration factor. The evaluation of the first term of (9) involves the determination of the phases of the reflection coefficients Γ_i and Γ_j. A generally adopted alternative procedure consists in giving to such a term a zero value yet regarding it as affected by an indetermination, the mismatch uncertainty M_u. This indetermination is equal to the half difference of the extreme values of the range for every possible phase combination. The advantage of this method is that these extreme values are expressible, from (7), through the reflection coefficient modulus

$$M_{u,\min} = 10\log(1-\rho_i\rho_j)^2 \tag{10a}$$

$$M_{u,\max} = 10\log(1+\rho_i\rho_j)^2. \tag{10b}$$

Therefore, the error due to the mismatching σ_γ results in

$$\sigma_\gamma = \frac{M_{u,\max} - M_{u,\min}}{2}. \tag{11}$$

We can now determine the error on the power P_a available within the TEM cell and at the antenna placed in the anechoic chamber. Thus, from (4) and (5), denoting with σ the errors expressed in decibels, we have

$$\sigma_{P_i} = \sigma_m + \sigma_{\alpha-1} + \sigma_\beta + \sigma_\gamma \tag{12a}$$

$$\sigma_{P_r} = \sigma_m + \sigma_{\alpha'} + \sigma_{\beta'} + \sigma_\gamma \tag{12b}$$

where σ_γ is the mismatch uncertainty between the bidirectional coupler and the power sensor and σ_m is the error on the reading of P_c and P_d introduced by the power meter. From (3), nonfactorable, we obtain

$$\sigma_{P_a} = 10\log(1+(dP_i + dP_r)/(P_i - P_r)) \tag{13}$$

where dP_i and dP_r can be determined through (12a) and (12b). The error σ_E, which affects the knowledge of the electromagnetic field generated in the TEM cell, is then from (1)

$$\sigma_E = \tfrac{1}{2}\sigma_Z + \tfrac{1}{2}\sigma_{P_a} + \sigma_d. \tag{14}$$

In order to estimate the electromagnetic-field uncertainty in the anechoic chamber, we have to consider first the gain uncertainty. From (6), it follows that

$$\sigma_G = \tfrac{1}{2}\sigma_{P_A} + \tfrac{1}{2}\sigma_{P_B} + \sigma_R + \sigma_\lambda. \tag{15}$$

The indetermination on P_A can be determined noticing that

$$\frac{P_{g,A}}{P_{g,s}} = \frac{P_{g,A}}{P_{g,Z_0}} \cdot \frac{P_{g,Z_0}}{P_{g,s}} \tag{16}$$

being $P_{g,A}$ the power supplied to the antenna by the generator, $P_{g,s}$ the power supplied to the power sensor by the generator,

TABLE I

FREQUENCIES (MHz)	PARTIAL ERRORS (dB)									TOTAL ERROR (dB)
	$\sigma_{\alpha-1}$	$\sigma_{\alpha'}$	σ_β	σ_γ	σ_m	σ_{P_a}	σ_d	σ_Z	σ_D	σ_E
0.1	0.50	0.50	0.24	0.02	0.09	0.90	0.01	0.17	0.25	0.79
1	0.50	0.50	0.24	0.02	0.09	0.90	0.01	0.17	0.25	0.79
50	0.33	0.32	0.11	0.02	0.09	0.58	0.01	0.17	0.25	0.63
100	0.26	0.24	0.24	0.02	0.09	0.66	0.01	0.17	0.25	0.68
150	0.26	0.23	0.24	0.02	0.09	0.62	0.01	0.17	0.25	0.66
200	0.24	0.21	0.24	0.02	0.09	0.61	0.01	0.17	0.25	0.65
250	0.24	0.20	0.24	0.02	0.09	0.60	0.01	0.17	0.25	0.65

Necessary parameters for the overall error evaluation in the TEM cell: $\sigma_{\alpha-1}$ = indetermination on the direct coupling factor; $\sigma_{\alpha'}$ = indetermination on the inverse coupling factor; σ_β = indetermination on the power sensor calibration factor; σ_γ = indetermination due to the mismatching between the bidirectional coupler and the power meter; σ_m = indetermination on the power meter reading; σ_{P_a} = indetermination on the available power; σ_d = indetermination on the plane-upper wall distance; σ_Z = indetermination on the characteristic cell's impedance; σ_D = indetermination due to the field nonhomogeneity in the TEM cell; σ_E = indetermination on the electric-field strength level in the TEM cell. The term $\sigma_{\beta'}$ is not reported in the table since it coincides with σ_β.

and P_{g, Z_0} the power that the generator would supply to a standard impedance Z_0 load. From (9) and (16), we can write

$$10\log\left(P_{g,s}/P_{g,A}\right) = 10\log|1-\Gamma_g\Gamma_A|^2 + 10\log\left(1-|\Gamma_s|^2\right) + \\ -10\log\left(1-|\Gamma_A|^2\right) - 10\log|1-\Gamma_s\Gamma_g|^2. \quad (17)$$

Both the second and the third terms are known and represent the mismatch loss of the sensor and the antenna, respectively. The relative errors are known too. The error on the second term is included into the error σ_β of the sensor calibration factor. The error on the third term σ_δ, is determined evaluating the error on the antenna reflection coefficient. The mismatch uncertainty connected to the measure of the power P_A, $M_{u,A}$, is given by the two remaining terms, and from (10a) and (10b) we have

$$M_{u,A\,\min} = 10\log\left(1-\rho_g\rho_A\right)^2 - 10\log\left(1+\rho_s\rho_g\right)^2 \quad (18a)$$

$$M_{u,A\,\max} = 10\log\left(1+\rho_g\rho_A\right)^2 - 10\log\left(1-\rho_s\rho_g\right)^2 \quad (18b)$$

from which

$$\sigma_{\gamma,A} = \frac{M_{u,A\,\max} - M_{u,A\,\min}}{2}. \quad (19)$$

The error σ_{P_A}, relative to the measure of the antenna A power, is then given by

$$\sigma_{P_A} = \sigma_m + \sigma_\beta + \sigma_{\gamma,A} + \sigma_\delta. \quad (20)$$

Likewise σ_{P_B}, relative to the measure of the antenna B power, is

$$\sigma_{P_B} = \sigma_m + \sigma_\beta + \sigma_{\gamma,B} \quad (21)$$

where

$$\sigma_{\gamma,B} = \frac{M_{u,B\,\max} - M_{u,B\,\min}}{2} \quad (22)$$

and

$$M_{u,B\,\min} = 10\log\left(1-\rho_B\rho_s\right)^2 \quad (23a)$$

$$M_{u,B\,\max} = 10\log\left(1+\rho_B\rho_s\right)^2. \quad (23b)$$

The error σ_S on the power density in the anechoic chamber is then from (2)

$$\sigma_S = \sigma_G + \sigma_{P_a} + 2\sigma_R + \sigma_K. \quad (24)$$

TABLE II

FREQUENCIES (MHz)	PARTIAL ERRORS (dB)										TOTAL ERROR (dB)
	$\sigma_{\alpha-1}$	$\sigma_{\alpha'}$	σ_β	σ_γ	$\sigma_{\gamma,A}$	$\sigma_{\gamma,B}$	σ_m	σ_{P_a}	σ_G	σ_K	σ_S
250	0.24	0.20	0.13/0.24	0.02	0.04	0.03	0.09	0.64	0.25	0.15	1.04
300	0.24	0.20	0.13/0.24	0.02	0.06	0.01	0.09	0.62	0.25	0.15	1.02
400	0.25	0.19	0.13/0.24	0.02	0.08	0.01	0.09	0.64	0.26	0.15	1.05
500	0.24	0.20	0.13/0.24	0.02	0.17	0.03	0.09	0.66	0.32	0.15	1.13
600	0.24	0.20	0.13/0.24	0.02	0.04	0.02	0.09	0.62	0.25	0.15	1.02
700	0.24	0.20	0.13/0.24	0.02	0.03	0.01	0.09	0.61	0.24	0.15	1.00
800	0.24	0.20	0.13/0.24	0.02	0.06	0.03	0.09	0.67	0.26	0.15	1.08
900	0.24	0.19	0.13/0.24	0.02	0.05	0.02	0.09	0.65	0.25	0.15	1.05
990	0.24	0.20	0.13/0.24	0.02	0.10	0.03	0.09	0.67	0.28	0.15	1.10

Necessary parameters for the overall error evaluation in the anechoic chamber: $\sigma_{\alpha-1}$ = indetermination on the direct coupling factor; $\sigma_{\alpha'}$ = indetermination on the inverse coupling factor; σ_β = indetermination on the power sensor calibration factor; σ_γ = indetermination due to the mismatching between the bidirectional coupler and the power meter; $\sigma_{\gamma,A}$ = indetermination due to the mismatching between the generator and the antenna A; $\sigma_{\gamma,B}$ = indetermination due to the mismatching between the antenna B and the power meter; σ_m = indetermination on the power meter reading; σ_{P_a} = indetermination on the available power; σ_G = gain indetermination; σ_K = indetermination due to the reflection in the anechoic chamber; σ_S = indetermination on the power density level in the anechoic chamber. The term $\sigma_{\beta'}$ is not reported in the table since it coincides with σ_β.

The terms σ_R, σ_λ, and σ_δ, which appear in (15), (20), and (24) are negligible. The error σ_K, relative to the reflection coefficient and evaluated in conditions similar to those reported in [4], is $\leqslant 0.15$ dB.

It is useful to remember that the previously evaluated error in the TEM cell, given by (14), is not inclusive of a term. Such a term is the uncertainty σ_D, depending on the nonhomogeneity of the field, which has been evaluated to be $\leqslant 0.25$ dB for cells with a similar form factor [1].

The overall errors in far-field conditions and relative to the different frequencies, are reported in Tables I and II. In these tables, the basic parameters for the error evaluation are reported too.

It should be noted that the worst case method is regarded by some authors as too pessimistic [9], [10]. Indeed many of the errors associated with different instrumental chain segments, although systematic and not random, are independent of each other and therefore reciprocally combine randomly. On this basis, the use of the RSS (Root Sum of the Squares) method for the error evaluation should be justified; the use of such a method could approximatively halve the above reported overall errors expressed in decibels.

Further studies are necessary in order to evaluate the measurement errors due to the higher order modes and the loading effects in the TEM cell and the near-zone gain in the anechoic chamber.

V. Conclusion

The features of this laboratory could be proposed as an example for other laboratories which intend to operate in the field of prevention against the risks of RF and MW electromagnetic-field exposure, both as reference laboratories for hazard-probe calibration and for the evaluation of biological effects linked to particular experimental procedures. Indeed, we think that the standardization of the methods of these electromagnetic-field generation and control is an essential requirement to obtain results, which can be compared in each country.

In order to assure a continued comparison between laboratories, we realize the necessity of adopting a traveling standard for the measure of the RF and MW electromagnetic-field intensities. Furthermore, it should be desirable to adopt a system (dummy plus sensor) for the reciprocal comparison of dosimetric measurements.

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Electromagnetic Waves in Conical Waveguides with Elliptic Cross Section

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Abstract —**The electromagnetic field in a conical waveguide with an elliptical cross section is calculated with the aid of two scalar potentials which satisfy the Helmholtz equation, the Dirichlet, and the Neumann boundary condition, respectively. The transverse parts of the solutions of the Helmholtz equation in the sphero-conal coordinate system are products of periodic and nonperiodic Lamé functions. These functions allow a mode definition similar to that for conventional waveguides. Some transverse modal field distributions, together with the corresponding Lamé functions, are graphically represented for a special elliptic conical waveguide.**

I. Introduction

The electromagnetic field in the interior of a cone with an elliptical cross section can be built up by solutions of the Helmholtz equation in a similar manner as is done in the case of rectangular or circular waveguides [1], [2]. For these calculations, the sphero-conal coordinate system is used which has elliptic cones as coordinate surfaces.

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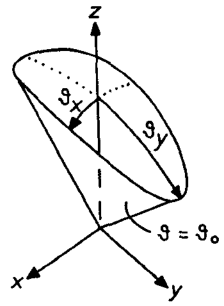

Fig. 1. Geometry of a cone with elliptic cross section.

The resulting modes show field configurations similar to those of modes in elliptic hollow pipes described by Chu [3]. Field lines of the lowest mode have already been given by Ng [4], but higher order modes have not been calculated as far as the authors know.

In this paper, only a short survey of the solution theory of the Helmholtz equation in sphero-conal coordinates and the involved Lamé functions is given. Details may be found in [4]–[11].

II. Solution of Maxwell's Equations in Sphero-conal Coordinates

The relation between Cartesian coordinates and the sphero-conal coordinates r, ϑ, φ can be defined by (1). In the special case $k^2 = 1$, these coordinates become the well-known spherical coordinates, with the z-axis being the polar axis

$$\begin{aligned} x &= r\sin\vartheta\cos\varphi \\ y &= r\sqrt{1-k^2\cos^2\vartheta}\,\sin\varphi \\ z &= r\cos\vartheta\sqrt{1-k'^2\sin^2\varphi} \\ &0 \le k, k' \le 1, \quad k^2 + k'^2 = 1 \\ &0 \le r < \infty, \quad 0 \le \vartheta \le \pi, \quad 0 \le \varphi \le 2\pi. \end{aligned} \tag{1}$$

The coordinates surfaces $\vartheta = \vartheta_0 = \text{const.}$ are cones with an elliptic cross section (Fig. 1). The extreme flare angles are

$$\vartheta_x = \vartheta_0$$

and

$$\vartheta_y = \arccos(k \cdot \cos\vartheta_0) \quad (\vartheta_y \ge \vartheta_x \text{ if } \vartheta_0 \le \pi/2). \tag{2}$$

The electromagnetic field in such a cone can be calculated with the aid of the substitution

$$\begin{aligned} \vec{H} &= \operatorname{curl}(\psi^E \vec{r}) && \text{for TM-waves and} \\ \vec{E} &= -\operatorname{curl}(\psi^H \vec{r}) && \text{for TE-waves, respectively.} \end{aligned} \tag{3}$$

Then Maxwell's equations demand that the scalar functions ψ^E and ψ^H must satisfy the Helmholtz equation

$$\Delta\psi^{E,H} + \kappa^2\psi^{E,H} = 0 \quad (\kappa\text{: wave number}). \tag{4}$$

In detail, (3) reads for TM-waves

$$\begin{aligned} E_r &= \frac{1}{j\omega\epsilon_0}\left[\frac{\partial^2(r\psi^E)}{\partial r^2} + \kappa^2 r\psi^E\right], && H_r = 0 \\ E_\vartheta &= \frac{1}{j\omega\epsilon_0 h_\vartheta}\frac{\partial^2(r\psi^E)}{\partial r\partial\vartheta}, && H_\vartheta = \frac{r}{h_\varphi}\frac{\partial\psi^E}{\partial\varphi} \\ E_\varphi &= \frac{1}{j\omega\epsilon_0 h_\varphi}\frac{\partial^2(r\psi^E)}{\partial r\partial\varphi}, && H_\varphi = -\frac{r}{h_\vartheta}\frac{\partial\psi^E}{\partial\vartheta} \end{aligned} \tag{5}$$

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